The reflection coefficient of a layer exhibiting high attenuation caused by interlayer flow

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Summary

The interlayer-flow model (1D White's model) is a physically based model for wave attenuation in the low-frequency range. We use the analytical solution of the interlayer-flow model, and a 1D analytical solution for the frequency-dependent reflection coefficient of an elastic layer embedded in an elastic medium, to study the reflection coefficient of a layer exhibiting high attenuation. We are specifically interested on the combined effect of (both frequency-dependent) attenuation (caused by the interlayer-flow) and tuning (constructive and destructive interferences of the waves reflected from the top and bottom of the layer). The result could be a significant reflection coefficient of a layer with high attenuation, when the real part of impedance contrast between the layer and the elastic background medium is negligible. I.e., reflection only due to attenuation and a geometric effect (the layer thickness). Our study of reflections from a layer exhibiting high attenuation caused by interlayer flow could be applied to hydrocarbon reservoirs.

Please see the following recent publication for more detail:
Low-frequency reflections from a thin layer with high attenuation caused by interlayer flow
Beatriz Quintal, Stefan M. Schmalholz, and Yuri Y. Podladchikov, Geophysics **74**, N15 (2009), DOI:10.1190/1.3026620

Introduction

Quintal et al. (2009) showed that, for a wide range of realistic petrophysical parameters for sandstones partially saturated with water and gas, the quality factor, Q, can be as small as 2 in the interlayer-flow model (White et al., 1975). They applied the interlayer-flow model to study the reflection coefficient of a thin (compared to the wavelength) layer that is partially saturated with water and gas, and exhibits such high attenuation. They showed that the reflection coefficient of the layer, caused only by contrast in attenuation between the layer and the non-attenuating background medium, can be larger than 10 % for $Q < 4$.

Besides the frequency-dependent behaviour of the attenuation in the interlayer-flow model, the reflection coefficient of a layer, R, presents a frequency-dependent behaviour related to the thickness of the layer, referred to as tuning effect, which results from constructive and destructive interferences of the waves reflected from the top and bottom of the layer (e.g., Kallweit and Wood, 1982). Both frequency-dependent behaviours can occur having their maxima coinciding, what can result in a significant reflection coefficient of a layer exhibiting high attenuation, but with a negligible real part of impedance contrast to the background medium. In this study we use the interlayer-flow model to investigate the combined effect of the frequency-dependent attenuation and tuning on the reflectivity of a layer.

The frequency-dependent Q of the interlayer-flow model

In the interlayer-flow model (White et al., 1975), a partially saturated rock is represented by two periodically alternating layers of media 1 and 2. Each layer is a fully saturated poroelastic solid that differs by the pore fluid properties. Attenuation and dispersion of the phase velocity are caused by wave-induced fluid flow, generated by pressure differences between the layers. The analytical solution for the interlayer-flow model yields the frequency-dependent quality factor, Q, and phase velocity, V_P, (Carcione and Picotti, 2006):

$$Q = \mathrm{Re}(E)/\mathrm{Im}(E)\,, \tag{1}$$

$$V_P = (\mathrm{Re}(1/V))^{-1}\,, \tag{2}$$

where E is the complex modulus for a P-wave traveling along the direction perpendicular to the layering, and V is the complex velocity, or

$$V = \sqrt{E/\rho}\,, \tag{3}$$

where ρ is the density of the partially saturated rock. Quintal et al. (2009) rearranged the equations of the analytical solution of the interlayer-flow model, defining E as the product of a real number, E_0, and a complex number, b,

$$E = E_0 b\,, \tag{4}$$

$$E_0 = \left(p_1/E_{G1} + p_2/E_{G2}\right)^{-1}, \tag{5}$$

$$b = \left(1 + (I_1 g_1 + I_2 g_2)^{-1}\right)^{-1}, \tag{6}$$

so that Q can be expressed as

$$Q = \mathrm{Re}(b)/\mathrm{Im}(b)\,. \tag{7}$$

The indexes 1 and 2 refer to the two different porous media, i.e., the two periodically alternating layers. For each saturated porous medium (j = 1, 2):

$$g_j = K_{Ej} \Big/ \left(2E_0 (r_2 - r_1)^2 p_j\right), \tag{8}$$

$$I_j = \sqrt{i\omega s_j}\,\coth\left(i\omega s_j / 2\right), \tag{9}$$

$$s_j = \eta_j d_j^2 / (K_{Ej} k_j)\,, \tag{10}$$

where $p_j = d_j/(d_1+d_2)$, and d_j is the layer thickness. The quality factor (equation 7) was written as a function of two groups of petrophysical parameters, s and g, and the angular frequency,

ω. Table 1 shows the symbols used for the basic petrophysical parameters, and the remaining parameters are defined in Table 2, with the index j omitted for clarity.

Table 1. *Symbols used for the basic petrophysical parameters.*

Symbol	Parameter
ϕ	Porosity
K_m	Bulk modulus of dry frame
μ_m	Shear modulus of dry frame
K_s	Bulk modulus of the grain
μ_s	Shear modulus of the grain
K_f	Bulk modulus of the fluid
η	Viscosity
k	Permeability
ρ_s	Density of the grain
ρ_f	Density of the fluid

Table 2. *Definition of some parameters.*

Definition	Parameter
$E_G = K_G + 4\mu_m/3$	P-wave modulus of the saturated rock
$K_E = E_m M/E_G$	Effective modulus
$r = \alpha M/E_G$	Ratio P-wave fluid tension to total normal stress
$K_G = K_m + \alpha^2 M$	Gassmann modulus
$E_m = K_m + 4\mu_m/3$	Dry-rock P-wave modulus
$M = \left(\frac{\alpha-\phi}{K_s} + \frac{\phi}{K_f}\right)^{-1}$	Solid-grain bulk modulus, where $\alpha = 1 - K_m/K_s$

In the interlayer-flow model, Q has always a minimum at a frequency referred to as transition frequency, f_{tr}. Only considering the value of Q at f_{tr}, and for a wide range of realistic petrophysical parameters for sandstones partially saturated with water and gas, Quintal et al. (2009) derived an expression for the optimal patch-size ratio (or saturation ratio) that provide the lowest value of $Q_{\min}$:

$$\left(\frac{d_2}{d_1}\right)^{\min} = \sqrt{\frac{E_{G2}K_{E2}}{E_{G1}K_{E1}}}, \tag{11}$$

where indexes 1 and 2 refer to water and gas saturation, respectively.

The reflection coefficient of a layer

The frequency-dependent reflection coefficient, R, can be calculated with a 1D analytical solution for an elastic layer embedded in an elastic medium (e.g., Brekhovskikh, 1980):

$$R = \frac{1-z}{1+z}\left(1 - \frac{4z\exp(-i2h\omega/V_l)}{(1+z)^2 - (1-z)^2\exp(-i2h\omega/V_l)}\right), \tag{12}$$

where h is the layer thickness, and $z = V_l\rho_l/V_b\rho_b$ is the impedance ratio. The subscript l refers to the layer, and b corresponds to the background medium. When the layer exhibits attenuation, V_l is complex, and therefore R. The magnitude of the reflection coefficient is the absolute value of R.

The combined effect of tuning and frequency-dependent attenuation

In our study, a layer exhibiting attenuation caused by interlayer flow is embedded in a non-attenuating background medium. We use equation 12 and the analytical solution of the interlayer-flow model (equations 1 to 10) to study the combined effect of the frequency-dependent attenuation and tuning (also frequency-related) on the reflectivity of a layer. We consider a model setup with no real part of impedance contrast between the layer and the background medium, only attenuation contrast caused by interlayer-flow in the layer. The densities of the background and of the layer are the same; and the velocity in the background medium is equal to the real part of the frequency-dependent velocity in the layer (which also contains a complex component, responsible for attenuation). I.e., for the velocity of the layer, V_l in equation 12, we use the complex velocity (equation 3) calculated with the analytical solution of the interlayer-flow model, and for the velocity of the background medium, V_b, we use the real part of that complex velocity (equation 2).

The petrophysical properties of the layer are given in Table 3 and correspond to sandstone partially saturated with water and gas. We calculate K_m and μ_m using Pride's relations (Pride, 2003) with c = 48 (poorly consolidated sandstones), and we choose a heterogeneity size, d_1+d_2, of 0.48 m. We use equation 11 to calculate the saturation ratio (or patch-size ratio) that optimizes the minimization of Q for our set of parameters, which corresponds to 9 % of gas saturation in the layer. I.e., the partially saturated medium is composed of periodically alternating layers saturated by gas and water, the gas-saturated layers being 0.04-m thick, and the water-saturated ones, 0.44-m thick. For this medium with attenuation caused by interlayer flow, $Q_{\min}$ = 3.5 and f_{tr} = 8.5 Hz.

Table 3. *Petrophysical parameters used in this study.*

Rock matrix	Sandstone	
K_s (GPa)	36	
k (mD)	300	
ρ_s (kg/m^3)	2650	
ϕ	0.35	
Fluid constituent	Water	Gas
K_f (GPa)	2.4	0.022
ρ_f (kg/m^3)	1000	100
η (Pa s)	0.001	10^{-5}

Using equation 12, we calculate the reflection coefficient of the layer described above, due to only attenuation contrast caused by interlayer-flow in the layer, as a function of h/λ and f/f_{tr}, where λ is the wavelength and f is the frequency (Figure 1). We see that the maximum value of the reflection coefficient, for our set of parameters, is 11.5 %, which occurs at the transition frequency (when $f = f_{tr}$), and when the layer thickness, h, is approximately $\lambda/4.3$, i.e., when the effect of attenuation combined with the effect of tuning is optimal.

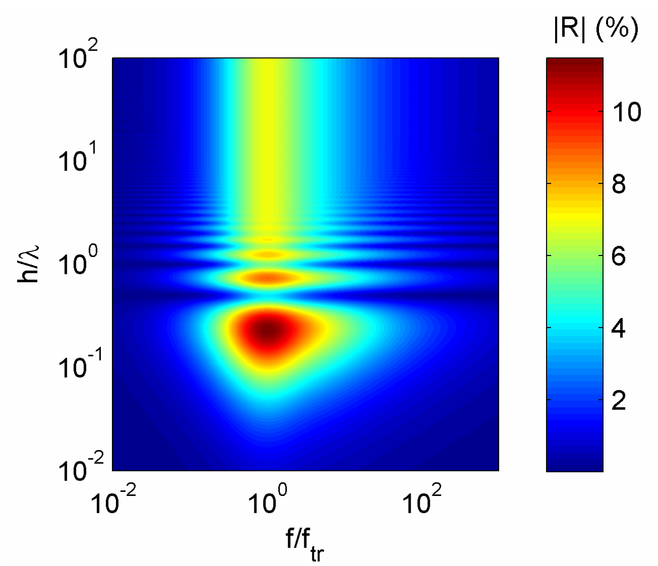

Figure 1. *Absolute value of the reflection coefficient of a partially saturated layer, only due to attenuation caused by interlayer flow within the layer.*

Discussion and conclusions

High values of attenuation in hydrocarbon reservoirs have often been observed at seismic frequencies (Chapman et al., 2006), and the major cause of seismic wave attenuation in porous media is presumably wave-induced fluid flow on the mesoscopic scale (White et al., 1975; Carcione and Picotti, 2006). We therefore apply our study of reflections from a layer exhibiting high attenuation, caused by interlayer flow (wave-induced fluid flow between layers), to hydrocarbon reservoirs.

Quintal et al. (2009) showed that for partial water/gas saturation, the highest attenuation occurs for a small amount of gas (about 9 %). Zones of small gas saturation and high attenuation, for example, may be located around the hydrocarbon reservoir at the water/gas or oil/gas contacts where the saturation changes gradually. Goloshubin et al. (2006) showed that hydrocarbon reservoirs exhibit increased reflective properties at low frequencies, and that expanding the active seismic bandwidth to low frequencies has a strong potential for predicting fluid content. Our study shows that when the elastic (real part of) impedance contrast between a layer with high attenuation and the surrounding medium is negligible, the layer might be "invisible" in a certain frequency range, but it might exhibit significant reflectivity at another frequency at which its attenuation is high. A variation of one order of magnitude in the transition frequency, f_{tr}, can easily occur, since it is directly proportional to the permeability, k (Dutta and Seriff, 1979; Quintal et al., 2009). Variations of that order are observed in the permeability of commercial hydrocarbon reservoirs (normally from 100 mD to 1 D).

In our example, the combined effects of frequency-dependent attenuation and tuning yielded a reflection coefficient of 11.5 % for a layer exhibiting high attenuation (Q = 3.5), and with no real part of impedance contrast between the layer and the surrounding elastic medium.

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